Load Response on a Large Suspension Bridge during the NYC Marathon Revealed by GPS and Accelerometers

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INTRODUCTION

Global Positioning System (GPS) geodesy is a well-known tool to measure the long-term surface deformation of the Earth at various temporal and spatial scales, and it is widely used in studies of geodynamics. GPS dual-frequency geodetic-quality receivers are typically sampled at rates of 15 to 30 s and analyzed in a batch mode over intervals from one hour to 24 hours. Recently, 1-Hz or greater rate sampled continuous GPS has been recognized as useful in combination with seismometers to capture the long-period seismic waves from large earthquakes and to detect volcano inflation and tsunamigenic landsliding (Bock *et al.* 2000; Larson *et al.* 2003; Bock *et al.* 2004; Mattia *et al.* 2004). Modern GPS receivers are capable of sampling at rates as high as 50 Hz (Genrich and Bock 2006).

The impact of heavy pedestrian movement on a highway suspension bridge has not yet been documented by precise observations, nor has it been modeled, although the impact of vehicles and wind was observed and used to constrain theoretical analysis (Nakamura 2000; Roberts *et al.* 2004; Guo *et al.* 2005). A large crowd can gather on a highway suspension bridge under extreme natural or human-generated conditions like earthquakes or power blackouts (Julavits 2003); an unpredicted response to the crowd was observed on a footbridge (Strogatz *et al.* 2005). Using GPS observations, here we evaluate the deflection of a large suspension bridge that connects Brooklyn with Staten Island at the mouth of upper New York Bay under the load of marathon runners and under the load of vehicles. Furthermore, by combining GPS and accelerometer data, we find that the runners excited the vibrations of the bridge at a modal frequency of 2.8 Hz, vastly different from the first vertical frequency of 0.13 Hz excited by vehicles. Our results provide a new constraint on theoretical models since the deflection caused by a distributed crowd is sensitive to the bridge properties untested by a conventional concentrated load such as a truck. These new observations can help predict a bridge's response to very-long-wavelength disturbances that might arise during earthquakes (Allam and Datta 2002; Smyth *et al.* 2003). Additionally, we show that GPS observations can be efficiently analyzed in real time using individual data epochs (Bock *et al.* 2000) with the same precision as multiepoch kinematic methods requiring postprocessing.

GPS SYSTEMS AND ACCELEROMETERS ON THE BRIDGE

We deployed 5-Hz GPS instruments and 100-Hz force-balance accelerometers (FBA) on the suspension bridge to observe load response from human, traffic, and environmental loads, both static and dynamic. Our experiment was motivated by the New York City Marathon, which has started every year since 1976 in Staten Island, with the first two miles almost entirely on the bridge (the marathon was within the confines of Central Park in 1970–1975). More than 30,000 marathon runners crossed the bridge within about one-half hour on 7 November 2004. We pursued two goals: 1) to assess the response of the bridge to a variety of factors (sudden impact of runners, heavy traffic, ambient temperature, and wind); and 2) to elucidate an optimal combination of both methods, GPS and FBA, for structural response analysis (Smyth and Wu 2007).

We started observations before the marathon on 6 November 2004 at 00:00 EST and finished on 7 November at 16:00 EST. Regular traffic was stopped on 7 November from 00:00 EST to about 11:00 EST. The marathon runners were on the bridge on 7 November between about 10:00 and 10:40 EST.

We used three GPS systems, each consisting of the Thales Navigation microZ-CGRS dual-frequency receiver and the Dorne–Margolin antenna with choke rings. The 128-MB solid-state memory in the receiver allowed us to collect the data at a 5-Hz sampling rate for up to 42 hours without downloading. Two systems were deployed on the upper deck of the bridge, with one system at midspan on the south side of the deck, station ST, and the other 150 m to the west of midspan on the north side, station NT (figure 1). The third system served as reference and was installed 1.6 km away from midspan on top of a building, station RF. The antennas on the bridge were mounted on three-legged aluminum-braced geodetic monuments, with the top 3 m above the roadbed to avoid obstructions of the sky caused by moving vehicles. The azimuth of the bridge is 71° so that the east

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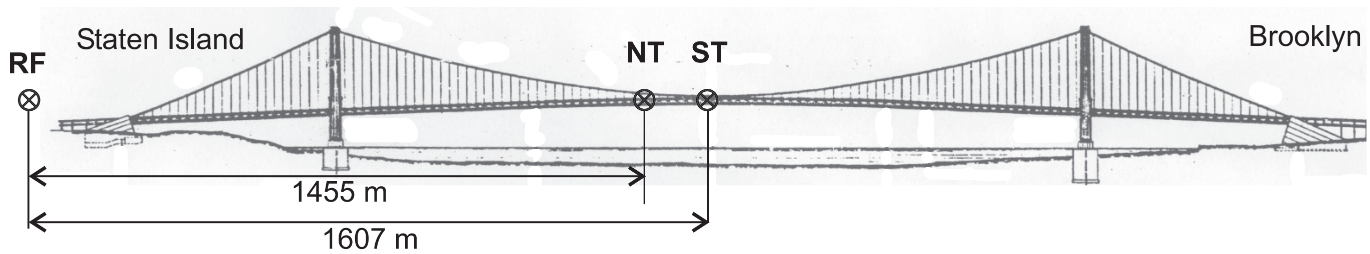


▲ **Figure 1.** Location of GPS antennas on the bridge. GPS antennas were installed at stations NT (northern truss), ST (southern truss), and RF (Staten Island). Observations at NT and ST were processed with respect to RF.

component of the GPS-estimated vector between the reference and bridge antennas is approximately aligned along the bridge, and the north component is transverse to the bridge.

At the same locations on the bridge as GPS, we installed two Kinematrics EpiSensor accelerometers set to 100-Hz sampling. A third accelerometer was installed on top of the western bridge tower. The EpiSensor is a tri-axial FBA-type strong-motion instrument that is designed to stay on scale during strongest seismic shaking.

With respect to measuring displacements of a structure, FBA is a high-pass filter, while GPS is a filter with flat response. For example, FBA, but not GPS, is highly sensitive to small displacements as long as they are rapid, such as the vibrations of the bridge at its natural frequencies with submillimeter amplitude excited by the rhythm of marathon runners. In contrast, GPS, but not FBA, can easily capture slow deflections such as those caused by the load of runners occupying the bridge, as long as the amplitude is above the instrumental noise level of 1–3 mm. It follows that a combination of GPS and FBA can vastly enhance the range of observed deformations of the structure.

KINEMATIC PROCESSING OF HIGH-FREQUENCY GPS OBSERVATIONS

The GPS observations were processed as hourly sessions, first by the RTD instantaneous positioning software (Bock *et al.* 2000, http://www.geodetics.com) and at a final stage by the TRACK software designed to process aggregated data sets (Herring 2005; King and Bock 2005). Both methods provided closely matching kinematic solutions for antenna positions (figure 2). RTD was run automatically in simulated real-time mode while TRACK required significant hands-on post-processing to correctly fix integer-cycle phase ambiguities. Even though TRACK was more robust with respect to multipath and signal diffraction caused by bridge elements, the utility of RTD instantaneous positioning was established as a viable precise tool for real-time structural monitoring and engineering seismology applications.

With the distance between the reference station and stations on the bridge as short as 1.6 km, we could ignore atmospheric (ionospheric and tropospheric) delays, and hence improve the baseline vector estimates by a factor of 3–5 compared to medium-distance applications (Bock *et al.* 2000). The major source of error in GPS observations on the bridge is multipath, which can reach in the phase observable ~ 50 mm with periods of several minutes (Leick 1995). We applied two methods to suppress the multipath error: 1) differencing the time series of computed kinematic positions separated by a sidereal day equal to 23h 56m 04.1s (Genrich and Bock 1992); and 2) high-pass filtering of the time series. Method 1 is useful when there are slow deflections on the current day (like the marathon day) but not on previous days. Method 2 is quite efficient when the goal is to study the oscillations of the bridge with periods less than the multipath corner period (3 min).

We next characterize the response of the bridge to the runners, to the traffic, and to the ambient temperature.

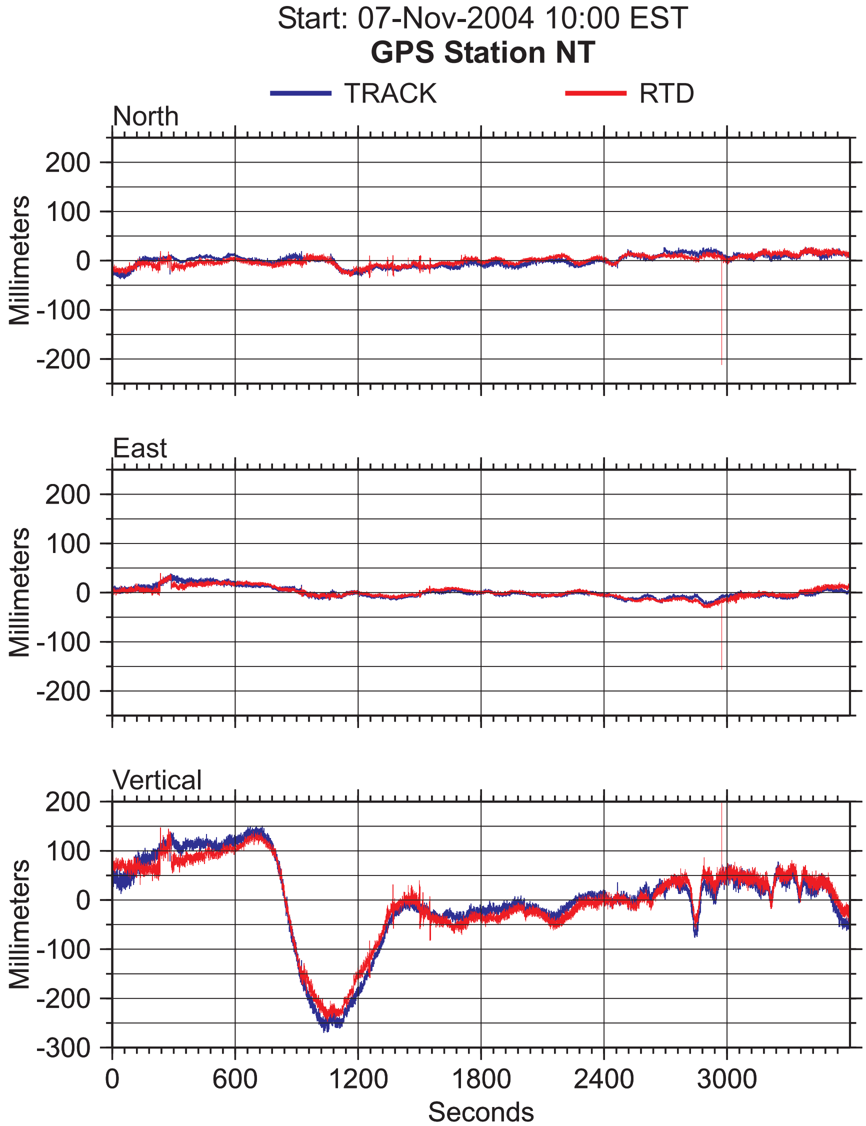
IMPACT OF RUNNERS AND TRAFFIC

At NT, the deck started to sag under the load of runners at 10:12 EST and reached the maximum deflection of about 350 mm in 5–6 min, at 10:17 (figure 3). Rapid recovery of 70% of the maximum deflection took 6 min and ended at 10:23 EST. A complete recovery to the original vertical position prior to the marathon was much slower and took about 27 min. The maximum deflection at ST (midspan) was smaller, approximately 250 mm. All phases of the deflection at ST (rapid sag, rapid partial recovery, and slow complete recovery to the original position) followed NT with a lag of 2 min, not surprising since runners moved from the west (Staten Island) and approached NT first (figure 1). Three rapid negative spikes in the vertical position of both stations at the end of the time series were caused by trucks crossing the bridge after the marathon (at about 10:48, 10:53, and 10:55 EST).

Disturbances in horizontal components of GPS positions at both NT and ST were much smaller, 10–30 mm (figure 3). We consider such correlated variations in position as representing the real bridge motion. Horizontal deflections were probably caused by non-uniform across-bridge distribution of runners on individual lanes. Wind, an alternative possible cause of horizontal fluctuations, was mild (8–10 km/h).

The start of the marathon excited rapidly growing vibrations of the deck captured by FBA (figure 3). The maximum vertical acceleration, with amplitude in excess of 200 mm·s^{-2}, corresponds in time to the sagging and subsequent recovery of the initial position of the deck. In terms of displacement, the vibration excited by runners was less than 1 mm.

A question arises, does the distribution of displacements along the length of the main span, predicted by a simplified ide-



▲ **Figure 2.** Kinematic GPS solutions for the marathon by two methods: RTD and TRACK. Uncorrected for multipath. The RTD instantaneous positions, obtained by replaying the data in a real-time scenario, contain 36 solution epochs out of 18,000 that were flagged on-the-fly as outliers (seen as spikes in the figure). For the sake of a complete dataset obtainable only in after-the-fact post-processing, the analysis described in this paper was performed using TRACK. However, the utility of the RTD software was established as a viable, precise, real-time technology for bridge and related monitoring applications.

alization using second-order deflection theory, agree with GPS observations? A simple theoretical model of the suspension bridge assumes that the bridge deck is an elastic beam held by vertical suspenders, which are steel cables connected to the main cable. The main cable is relatively flexible, it is supported by towers, and it is anchored to the ground at its ends. It should be noted that this basic model ignores the fact that the main span is attached to side spans through the cable and tower support system. Advanced mathematical models of linear and nonlinear modal bridge properties were stimulated by the collapse in 1940 of the Tacoma Narrows suspension bridge, Washington, USA, caused by violent torsional oscillations (Lazer and McKenna 1990).

A simplified equation for η, vertical deflection of the deck at abcissa x aligned along the bridge, is (Podolny 1999):

$$EI\eta'''' = p + H_p y'' + H\eta'' \quad (1)$$

where

E = modulus of elasticity of stiffening-truss steel (206,850 MPa),

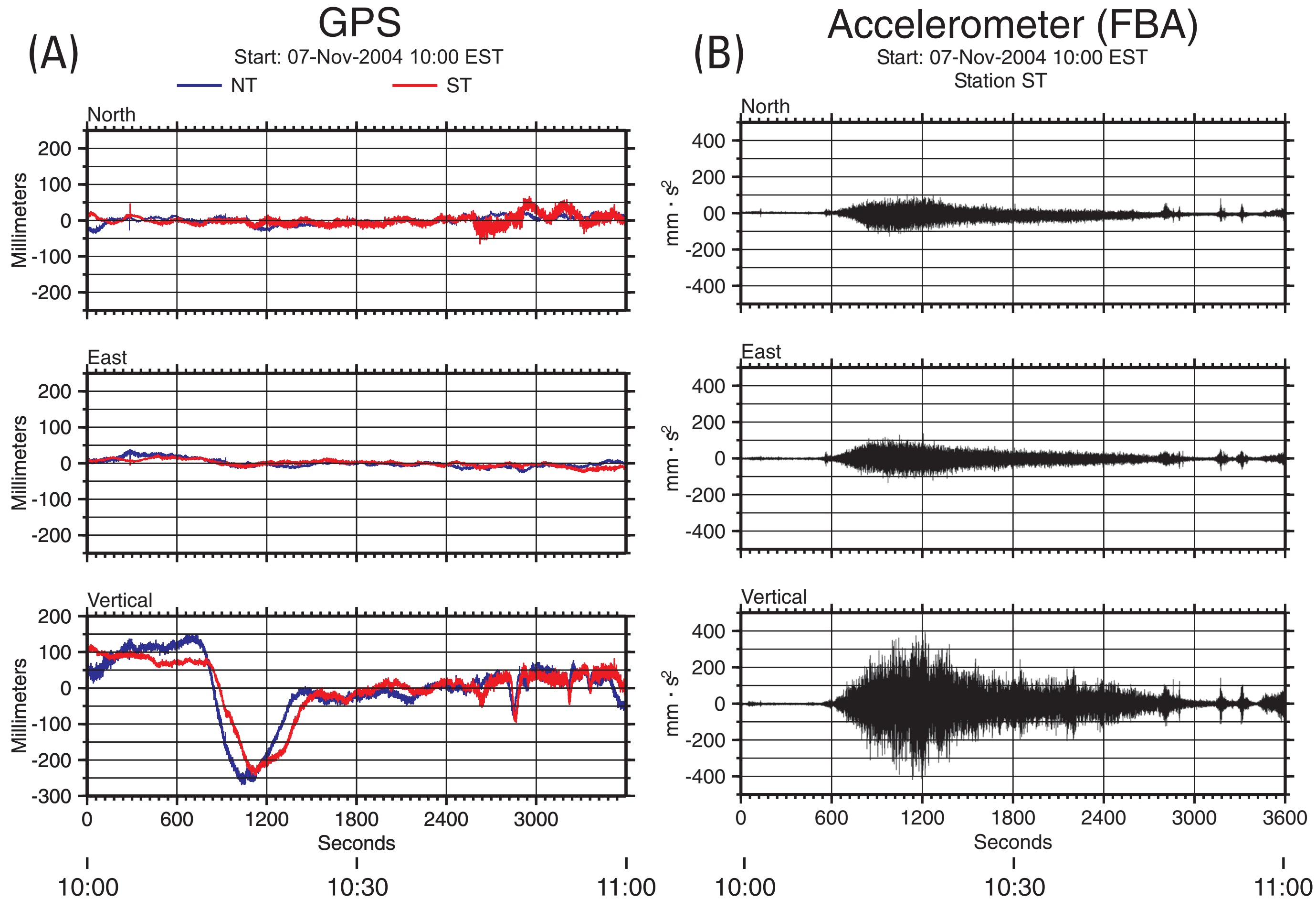
I = moment of inertia of deck cross-section (51.786 m^4),

H = horizontal component of cable tension under dead and live loads,

H_p = horizontal component of cable tension under total live load,

p = live load intensity per unit length,

y = vertical distance from cable support on the tower at a point where η is measured, and derivatives are taken with respect to x.



▲ **Figure 3.** A: Three components of deflection of the bridge deck under load of runners from GPS solution. Corrected for multipath. B: Three components of vibrations of the deck excited by runners, measured by the force-balance accelerometer (FBA).

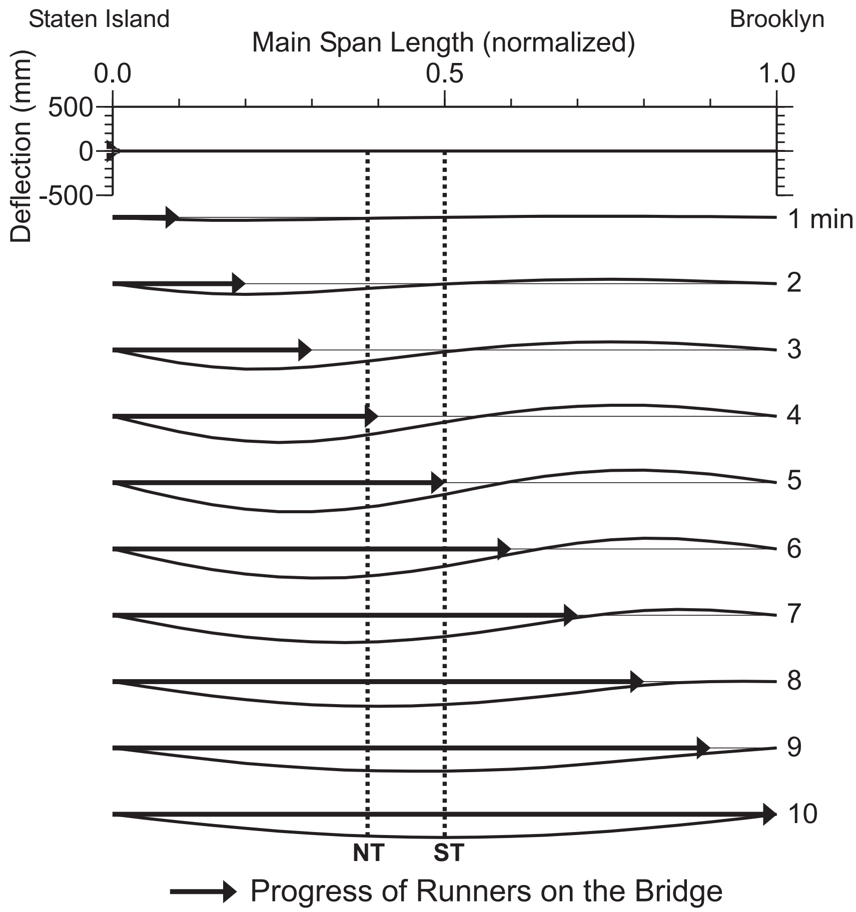
The solution of equation 1 by the iterative Fourier method (Timoshenko and Young 1965) explains well the evolution of deflection with time at NT and ST at a loading stage (figure 4). The best fit to the observed amplitude of sagging is achieved for a value of uniform live load $p = 4.4$ kN/m (0.3 kips/ft). This value of p reflects a scenario of runners occupying all six lanes of the upper deck, with the average weight of a runner 64 kg (140 lbs) and the average spacing between runners along the lane 0.85 m (2.8 ft). The worst-known parameter is I, whose value cannot be simply associated with the moment of inertia of the stiffening truss. The double-deck cross-section has a stiffening truss connecting the upper and lower roadways, transverse beams, stringers, and cross-frames such that it behaves as a stiff rectangular tube (Podolny 1999). A ratio of maximum deflections at both stations is sensitive to a value of I, with a best-fitting value of 52 m^4 (6,000 ft^4). Perhaps the two most interesting facts revealed by the theoretical solution are: 1) the maximum deflection occurs at quarter-span; and 2) deflection of the fully loaded bridge is less than the maximum observed at partial loading.

The phase of unloading the bridge at the end of the marathon is more complicated because the number of runners per lane per unit length and the number of occupied lanes was changing at this phase. Clearly, the unloading was slower than the loading as evidenced by both GPS and FBA (figure 3).

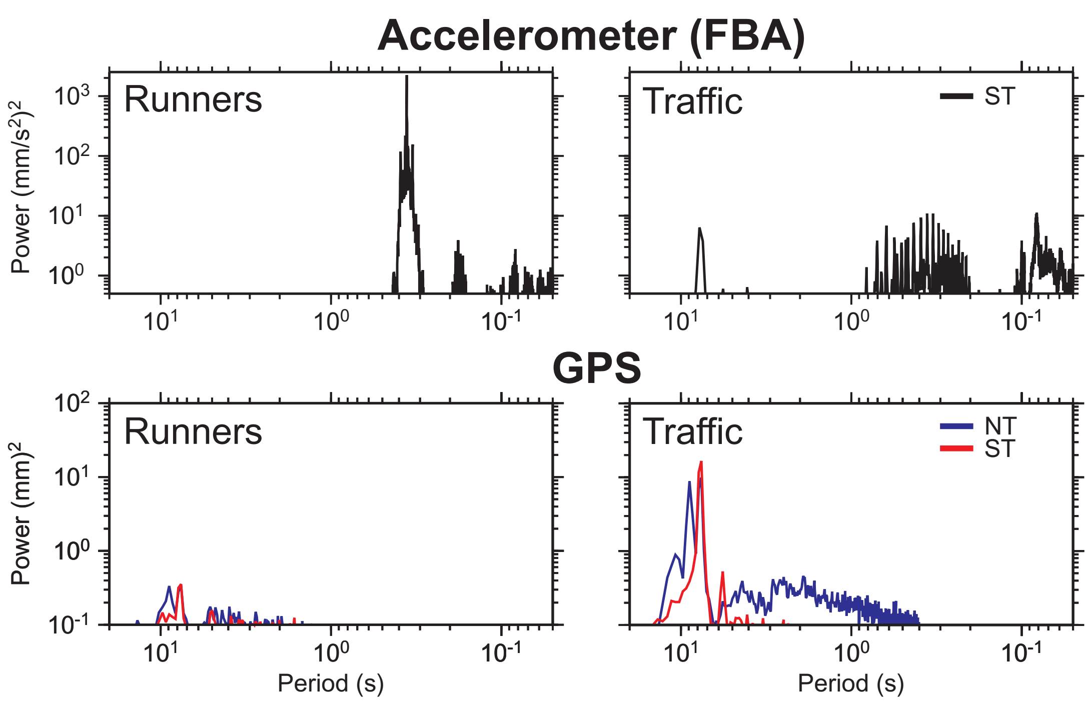
RESOLVING POWER OF GPS AT HIGH FREQUENCIES

It is reasonable to ask, what is the smallest amplitude of oscillation of the bridge deck that can be resolved by GPS? For shorter periods, less than 1 min, the limiting factor is the measurement noise with periods about 0.5 s and the 1-sigma variation of 2–3 mm in all three components at ST (see also Langbein and Bock 2004). Station NT is noisier at such short periods. The limiting factor to resolve an amplitude of the "slow" oscillation (periods longer than 2 min) is the residual multipath: 4–6 mm in horizontal components, 15 mm in the vertical.

The specific resolving capacity of GPS and FBA in a wide range of periods is illustrated with the power spectra of time series collected at midspan during the marathon and during intense regular traffic the day before (figure 5). A peak in the vertical spectrum of GPS and FBA observations with a period of 7.7 s, both at NT and ST, is persistent when the traffic is on



▲ **Figure 4.** Evolution of predicted vertical deflection of the deck during the period of loading by runners at various points on the bridge. For each vertical cross-section, the time elapsed since the beginning of loading is labeled on the right. Prediction is based on the second-order deflection theory. The best fit with observed deflections is achieved using the uniform live load 4.4 kN/m (p = 0.3 kips/ft).



▲ **Figure 5.** Power spectra of vertical oscillations of the deck from observations with force-balance accelerometers (FBA) and GPS.

the bridge and is notably inconspicuous during the marathon. The power spectra of both horizontal components are much smaller in magnitude than the vertical component. The higher-frequency modes captured by the accelerometer are also quite different during the marathon as compared with the modes excited by the traffic: an isolated peak at a period of 0.36 s (frequency 2.8 Hz) dominates during the marathon yet is inconspicuous when the bridge is occupied by traffic. Remarkably, this period is close to the beat rate of runners, about 0.4 s assuming a speed of 10 km/h and a step length of 1 m. We argue that the vibration excited by runners is associated with the natural frequencies of the bridge since its period is remarkably stable throughout the whole marathon, with variations less than 0.3% (compatible with an error of the estimate). Fortunately, the phase of steps of individual runners is incoherent over a distance spanning several runners, so that a small proportion of runners who step in phase can excite resonant oscillations at only a small amplititude.

GPS and FBA can also be compared in the time domain after conversion of GPS displacements to accelerations with appropriate filtering. Both the amplitude and phase of the 7.7-s mode agree well between the FBA and GPS (figure 6).

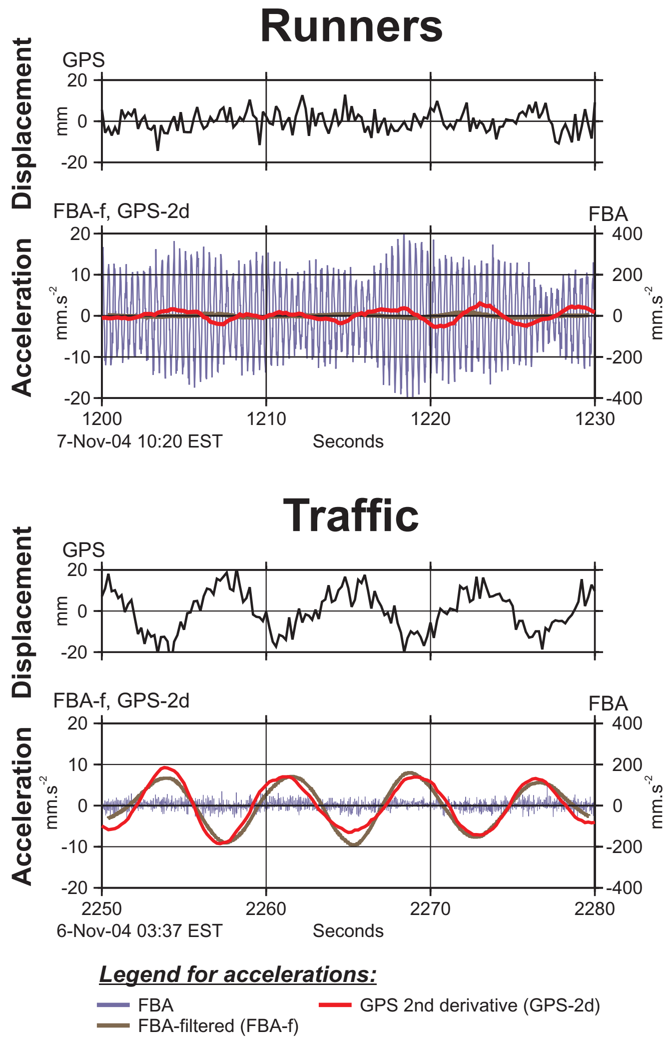
OPTIMAL FUSION OF GPS AND ACCELEROMETER DATA

To best capitalize on the displacement and acceleration measurement redundancy at the midspan of the bridge, it is possible to use a recently developed data processing technique (Smyth *et al.* 2006; Smyth and Wu 2007) that achieves a fusion of the information contained in each signal to yield improved displacement and velocity estimates. The procedure, which involves a multirate formulation of Kalman filtering and smoothing on the state-space measurement equation, also suppresses measurement noise, given estimates of the noise covariance. A comparison segment of just over 40 seconds of the GPS-measured midspan vertical displacement versus the displacement estimated by data fusion is shown in figure 7 for the marathon event. It is noteworthy that the procedure not only greatly reduces the GPS measurement noise, but it clearly injects the 2.7-Hz vibrations seen in the FBA data (corresponding to displacement oscillations of only 1–2 mm) into the displacement data. This is particularly interesting because the 2.7-Hz frequency content is above the 2.5-Hz Nyquist frequency of the GPS sampling.

THERMAL DEFORMATION OF THE BRIDGE FROM GPS DATA

The slowest variations in the vertical position of the bridge deck revealed by GPS occur at periods of several hours, showing strong correlation with air temperature (figure 8). Estimation by linear regression gives factors of –22 and –25 mm/°C at NT and ST respectively.

Theoretical temperature dependence of a sag in the suspension cable can be approximately estimated from the formula for a parabolic cable shape (Podolny 1999):



▲ **Figure 6.** Oscillations associated with the natural frequencies of the bridge, excited by runners and by traffic. Displacements are shown as observed with GPS. Accelerations are shown in three manners: 1) observed with FBA; 2) FBA low-pass filtered with a corner period of about 1 s; and 3) the second derivative of GPS displacements.

$$S = L\left[1+\frac{8}{3}\left(\frac{f}{L}\right)^2\right] \tag{2}$$

where

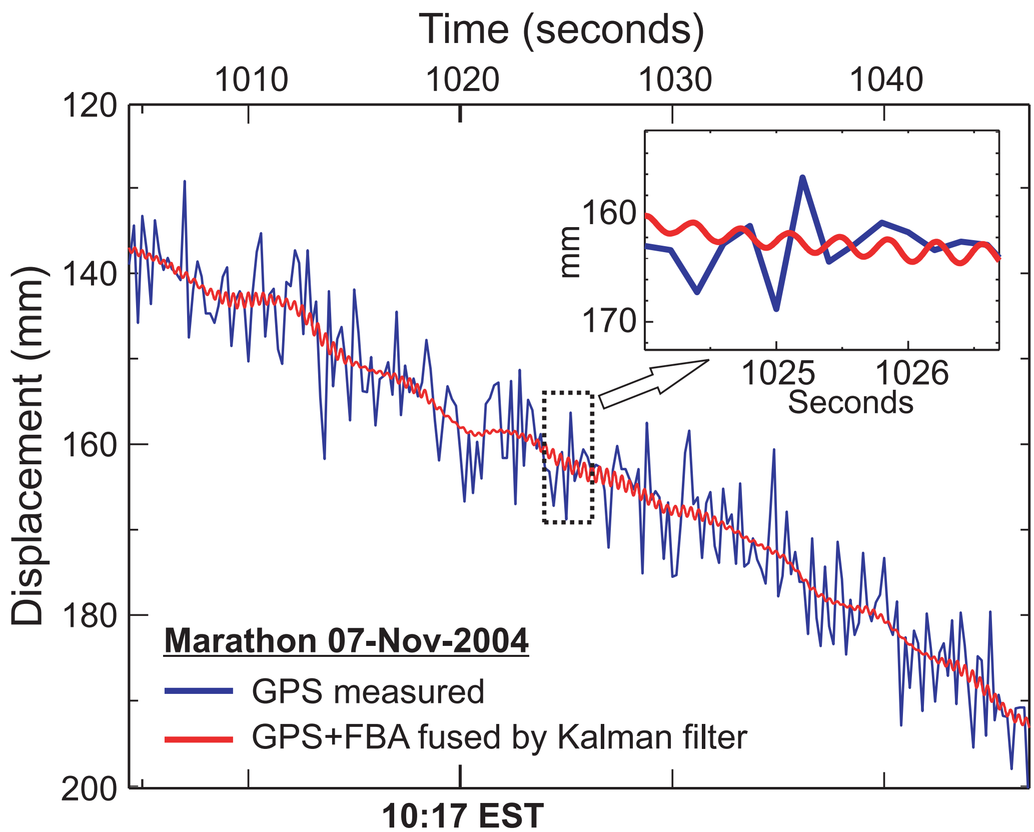
L = length of main span (1,298 m),

S = length of suspension cable between towers, and

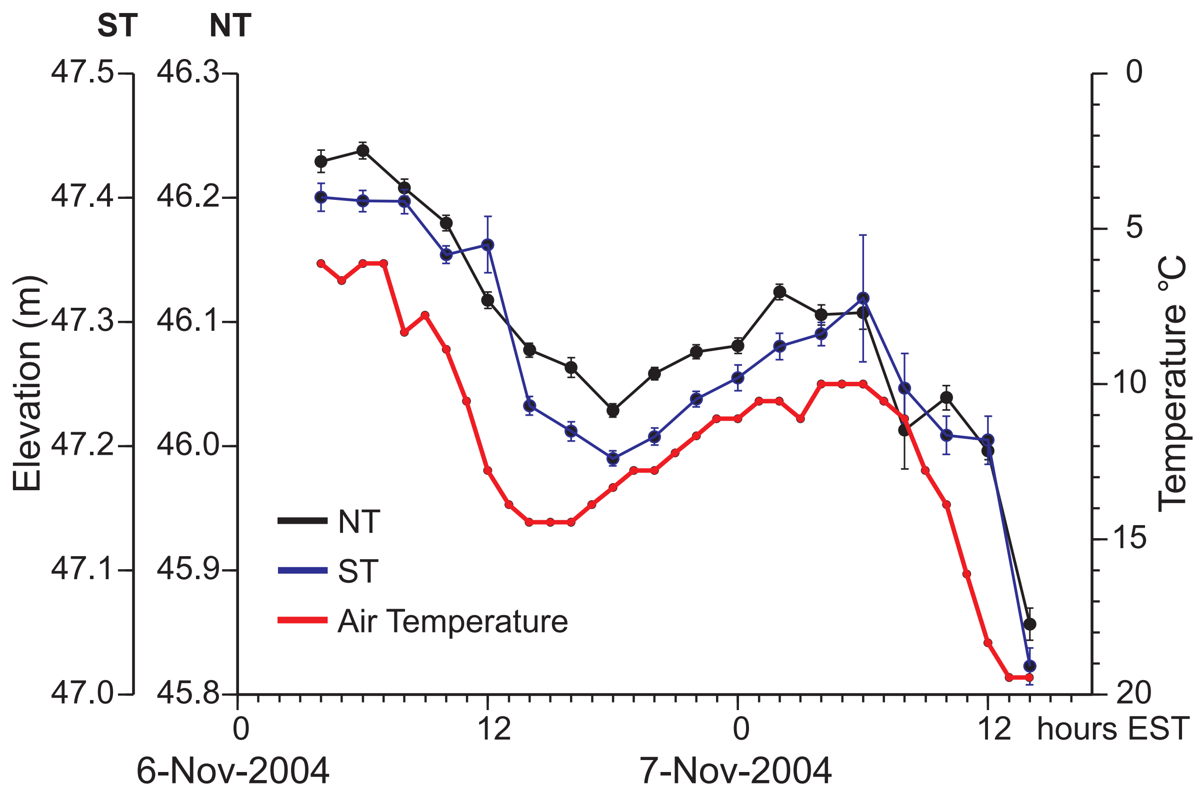
f = sag of cable (124 m).

We find the temperature dependence of the sag equal to –31 mm/°C. This factor must be corrected for the temperature change in the height of the tower, equal to +3 mm/°C. Hence the observed diurnal variation in the bridge deck elevation at midspan can be explained by the thermal behavior of the suspension cables and towers.

Deviations of the vertical deflection from linear dependence on temperature are less than 0.1 m; therefore the additional deflection caused by the traffic does not exceed that value.



▲ **Figure 7.** GPS measured displacements, and GPS + FBA displacements fused by Kalman filter.



▲ **Figure 8.** Variations of one-hour averaged elevations of NT and ST, and variations in one-hour averaged air temperature. Hourly GPS solutions were performed by GAMIT (King and Bock 2005) with respect to station RF (figure 1).

CONCLUSION

Our results demonstrate that a combination of 5-Hz GPS and 100-Hz force-balance accelerometers (FBA) on the bridge allowed us to estimate the response of the bridge to such factors as marathon runners, regular traffic, and air temperature in the range of periods impossible to span by GPS or FBA alone. For example, the vibrations excited by runners are small in terms of displacements, yet large in terms of accelerations; therefore FBA is efficient in contrast to GPS. On the other hand, the slow deflection under a load of runners is large in terms of displacements while the corresponding accelerations are very small; therefore GPS is efficient but FBA is not. Both methods, however, have adequate sensitivity to capture the symmetric vertical vibrations of the bridge deck at a period of 7.7 s.

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