Epoch-by-Epoch™ Real-Time GPS Positioning
in High Dynamics and at Extended Ranges

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ABSTRACT
There are many civilian and military applications requiring precise relative positioning. Geodetics, Inc. has demonstrated centimeter-level position solutions using their Epoch-by-Epoch™ (EBE) technology. EBE technology provides computational algorithms for instantaneous differential GPS processing of raw GPS measurement data (pseudorange and carrier phase) from one or more base stations and one or more rovers. EBE has been shown to have significant advantages over conventional GPS Real-Time Kinematic (RTK) algorithms in several ways, including 1) no initialization or re-initialization delays, 2) extended ranges over which dual-frequency GPS receivers can provide precise positioning, and 3) graceful degradation when a full set of measurement data is not available.

This paper will provide empirical data that was gathered during a test program, sponsored by the Office of the Secretary of Defense, to assess the performance in real time of EBE technology on realistic ARDS maneuvers.

INTRODUCTION
The Central Test & Evaluation Investment Program (CTEIP), which resides within the Office of the Director, Operational Test and Evaluation / Resources and Ranges (DOT&E/RR) within the Office of Secretary of Defense (OSD), has been funding a program to evaluate the performance of EBE technology for test and evaluation instrumentation applications.¹ During the program, Geodetics integrated its EBE software package with a number of commercial and SAASM-based receivers. These receivers were then tested in both live and simulated tests, under strenuous environments, including high dynamics and extended range. The results of these tests were then analyzed. The first part of the analysis focused on position solutions utilizing pseudorange and carrier phase measurements at high dynamics.

This paper begins with a background discussion of T&E applications requiring precision GPS. This is followed by a description of the EBE technology, EBE test results for three simulated high-dynamic maneuvers, and a summary that includes future work.

¹ This is the second of two papers written on EBE assessment. See [Macdonald, 2001] for the first paper.
T&E Applications Requiring Precision GPS — In the 1970’s, OSD recognized that use of GPS-based TSPI, i.e., tracking, offered excellent advantages over conventional TSPI instrumentation. In fact, GPS offered the potential to support an unlimited number of participants in a ‘world wide range’. In 1979, OSD conducted a major study to evaluate the potential of GPS as a TSPI source for the test and training range communities. The survey evaluated TSPI requirements at 22 major ranges of the three Services. GPS was evaluated considering such factors as real-time and post-mission TSPI and scoring accuracy, supportable vs. required data rates, number of participants, and coverage area. The OSD final report was submitted on 31 December 1982. The report concluded that GPS could be used as a cost-effective real-time TSPI source for 95 percent of all TSPI requirements.

Range Applications Program (RAP). As a result of this report, a tri-Service program was initiated to develop the assets for the real-time GPS-based positioning of low, medium, and high dynamic platforms. These assets were to be put in place at the major DoD test and evaluation ranges. The training community initially deferred the introduction of GPS-based TSPI; they are now in the process of converting most of their major range instrumentation to GPS.

GPS has undergone significant hardware and software enhancements during the last ten years. The advances have primarily focused on miniaturization of the GPS package and improvement in the accuracy of the GPS solution. Nonetheless, RAP initial production equipment is now ten years old. It is now believed that GPS can cost-effectively address the remaining 5% of unmet TSPI requirements that were called out (in the above mentioned) 1982 OSD report. For that reason, EnRAP is being pursued to provide the next generation RAP instrumentation.

Enhanced RAP Program (EnRAP). The EnRAP program is a CTEIP initiative to upgrade the basic RAP system in four areas: 1) Enhanced data link spectrum efficiency and capacity, 2) Higher accuracy GPS/inertial TSPI, 3) Miniaturization of modules, and 4) Development of a plug-and-play architecture.

EnRAP provides TSPI for a number of land, sea and air platforms, with the most stressing environment being that of a fighter aircraft (~10 g’s). EnRAP has a stringent requirement to enhance the real-time accuracy of today’s GPS instrumentation to meet the test needs of advanced weapon systems. Test instrumentation must provide a factor of ten enhanced accuracy when compared to the system-under-test (SUT). This translates to EnRAP (level III) TSPI requirement of 30 cm real-time and 10 cm post-mission. It should be noted that the required TSPI performance must be provided real time over a range area that can extend some 200 miles. Based on the test results presented in this paper, EBE has the potential to meet this real-time requirement for the next generation TSPI suite of instrumentation.
Multi-Service Target Control System (MSTCS) Project. CTEIP is also funding the MSTCS project to develop the next generation target control system. MSTCS is a GPS-based system that will replace and upgrade the Gulf Range Drone Control System (GRDCS), which is the Air Force’s ground-based multilateration target control system (TCS) that operates at the Gulf Range, FL, the Drone Formation Control System (DFCS), which is the Army’s ground-based multilateration TCS that operates at White Sands Missile Range (WSMR), NM, and the Integrated Target Control System (ITCS), which is the Navy’s radar-based TCS that operates at Pt. Mugu, CA.

MSTCS has a requirement for formation control of airborne targets and auto landing of full-scale drones such as the QF-4. Precise real-time GPS would support more realistic presentations of closely spaced airborne targets, and would provide a cost-effective GPS-only instrumentation package for auto-landing. Real-time accuracy on the order of 30 cm is required. As opposed to EnRAP, however, precision TSPI is only warranted in low dynamic applications, i.e., formation flight, auto-landing. The MSTCS range of operations can extend 200 miles from the control center. Based on the test results presented in this paper, EBE is able to meet the requirements for the next generation MSTCS.

Joint Advanced Missile Instrumentation (JAMI) Project. The JAMI project is another key CTEIP-funded program. JAMI is developing modular instrumentation packages for missile applications. The JAMI package supports three functions: 1) Missile termination, 2) TSPI, and 3) Vector-Scoring.

The TSPI and Vector-Scoring will be accomplished with a GPS (and inertial) engine on both the missile and target. Although TSPI must be provided in real time, the accuracy requirement is moderate (several meters). The vector-scoring requirement, on the other hand is precise (< 30 cm), but can be accomplished post mission. Because post-mission processing is allowed, existing RTK algorithms can be used for this purpose. However, operational considerations and excellent performance data (discussed in this paper) suggest that EBE technology may be the preferred approach meeting the vector-scoring requirement.

Epoch-by-Epoch™ Technology
Geodetics, Inc. has developed a new class of instantaneous, real-time precise GPS positioning and navigation algorithms, referred to as Epoch-by-Epoch™ (EBE) [de Jonge et al., 2000, Bock et al., 2003]. Compared to conventional RTK, integer-cycle phase ambiguities are independently estimated for each and every observation epoch. Therefore, complications due to cycle-slips, receiver loss of lock, power and communications outages, and constellation changes are minimized. There is no need for the initialization period (several seconds to several minutes) required by conventional RTK methods. More importantly there is no need for re-initialization immediately following loss-of-lock problems such as occurs when a mobile GPS
receiver passes under a bridge or other obstruction, or loses satellite visibility during an aerial maneuver.

In addition, EBE provides precise positioning estimates over longer reference receiver-to-user receiver baselines than conventional RTK. This feature supports testing for long-range operations, for example, positioning aircraft landing on a ship at sea (i.e., the reference receiver is on the ship).

Precise GPS positioning data provided by EBE is also used to provide platform attitude data. This can be accomplished by multiplexing a single user receiver among three antennas on the platform, or by deploying three complete GPS systems. Only two antennas would be necessary if only two of the three attitude angles were desired. Attitude can be determined to an accuracy of about 0.05 degrees (antenna distances of 1.5 meters).

EBE requires the use of a minimum of two receivers, each of which is tracking a common set of five or more satellites and providing simultaneous dual frequency phase data. Normally one of the receivers is stationary but this is not a requirement. It is anticipated that EBE technology would support end game scoring between a missile and an airborne platform. In this case the two receivers would be located on highly dynamic platforms.

EBE has been proven utilizing dual frequency receivers and operating at distances of up to 50 km from the nearest base station in unaided mode, and up to 250 km in aided mode. Aided mode requires reasonable knowledge of a-priori position (meter level) such as provided by a coupled IMU. Additionally the EBE algorithms operate in a network environment and make optimal use of all GPS measurement data at each epoch, gracefully degrading the position accuracies when some measurement data are not available. Further, the system will make use of IMU system, compensating for outages when sight to the satellites is blocked. This results in a robust and more reliable system.

OSD has seen sufficient static and low dynamic performance data to be convinced that the EBE concept deserves serious consideration for test applications. A test program is required because the scenarios of interest typically involve high dynamic platforms, and there is minimal data for these scenarios.

Epoch-by-Epoch™ promises numerous benefits including:

- **Computationally efficient algorithms** that provide a position estimate based on a single epoch in several milliseconds. This allows the real-time position estimate to be computed on the user platform (assuming reference station data is sent to the user platform).
- **An initialization period is not required.** Since RTK requires some period of time (that can be measured in seconds to minutes) to perform ambiguity resolution, this is an important capability for
platforms that: Require high accuracy (e.g., for end game scoring); cannot see the satellites until launch; and have short flight duration.

- A reinitialization period following loss-of-lock is not required, unlike RTK, which needs to restart the integer-cycle phase ambiguity resolution process. This is another important capability because OSD is considering EBE for many high dynamic applications where loss of lock and loss of data are likely.

Currently, there are receivers in production that will support the EBE requirement for simultaneous dual-frequency code and phase observations. Hence, it is not envisioned that a receiver development effort would be required if the dynamic test results prove favorable. However, it must be mentioned that many of the GPS receivers in use by the test (and training) community today do not support this dual frequency requirement. Hence, those systems could not realize the maximum benefit.

**Test Setup**

The RTD ("Real Time Dynamics") software package with EBE positioning was tested using three high-dynamic simulations based on realistic ARDS maneuvers developed as part of the TSPI study. An omni-directional antenna simulation was utilized for each of the maneuvers. The maneuvers and antenna model were programmed into the Guided Weapons Evaluation Facility (GWEF) satellite simulator. The GWEF simulator is a model 2400 GPS Constellation Simulator from Interstate Electronics Corporation, and was phase calibrated using the procedures outlined in [Anthony et al., 2001]. The NovAtel OEM4 was selected as the test receiver (see Figure 1). For each maneuver, reference data were collected from both a near and far reference receiver. The near reference receiver was located at distances of less than 10 km from the rover. The far reference receiver was located at distances of 88-96 km from the rover. The tests involved comparing the RTD EBE output positions for the rovers to the 'true' simulator positions. Although conducted in "post-processing" the EBE analysis was conducted as if the data were collected in real time.

![Figure 1. Guided Weapons Evaluation Facility (GWEF)](image-url)
Test Maneuver Descriptions
The three selected maneuvers were: 1) Split S – Cuban 8 – Split S (~4 g’s), 2) Post Hole (~3.8 g’s) and 3) 9g Turns (~9 g’s). Each maneuver contained three segments; a 15 second straight and level segment followed by the maneuver followed by another 15 second straight and level segment. The maneuvers are shown graphically in Figures 2-4. The first two maneuvers provided sufficient data (visibility to 5 or more common satellites) for phase processing with ambiguity resolution. The third maneuver was analyzed in ‘graceful degradation’ mode since only 4 satellites were available for about 50% of the time, including the entire first 720° turn. In this mode, RTD uses only pseudorange measurements when the number of common satellites fall less than the minimum 5 required for EBE processing.

Figure 2. Rover flight path and dynamics of the “Split S-Cuban 8-Split S” maneuver with up to 4g’s of acceleration. The velocity was kept constant at 100m/s, while the acceleration and jerks were varied.

Figure 3. Rover flight path and dynamics of the “Post/Hole” maneuver with up to 4g’s of acceleration. The velocity is kept constant at 100m/s, while the acceleration and jerks were varied.
Rover flight path and dynamics of the “9g Turns” maneuver with up to 9g’s of acceleration. The velocity is kept constant at 100m/s, while the acceleration and jerks were varied.

Test results

Figures 5-10 compare the EBE rover position output with the ‘true’ simulated positions for each of the three maneuvers, for both ‘near’ and ‘far’ reference stations. Table 1 provides standard (mean, standard deviation) and robust (median, interquartile range) statistics for the deviations from truth in each of the experiments. In order to isolate and characterize outliers, we calculate the median and interquartile range for the deviations from truth, which are less affected by outliers than standard statistics. The interquartile range (IQR) is defined as the range of the middle 50% of the data. From experience with many real and simulated data sets, we select the outlier criteria to be 4 times the IQR (this criteria corresponds to approximately $3\sigma$, if the single-epoch solutions were normally distributed).

**Figure 5.** Near position errors for “Split S-Cuban 8-Split S” maneuver
Figure 6. Far position errors for “Split S-Cuban 8-Split S” maneuver

Figure 7. Near position errors for “Post Hole” maneuver

Figure 8. Far position errors for “Post Hole” maneuver
**Figure 9.** Near position errors for “9g Turns” maneuver in graceful degradation mode

**Figure 10.** Far Position errors for “9g Turns” maneuver in graceful degradation mode

<table>
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<tr>
<th>Component</th>
<th>Mean (m)</th>
<th>Std. Dev. (m)</th>
<th>Dev. w/o Outliers</th>
<th>Median (m)</th>
<th>IQR (m)</th>
<th>% Outliers &gt; 4 IQR</th>
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**Table 1.** Positional accuracy (deviation from truth) of the maneuvers
Summary
As shown in Table 1, EBE yields cm-level real-time accuracies (1 standard deviation), for the “Split S-Cuban 8-Split S” and “Post Hole” maneuvers where base station distances are up to 10 km. These accuracies are obtained after removing position outliers greater than 4 times the IQR. The “Split S-Cuban 8-Split S” maneuver has about 7% of easily detectable outliers, while the “Post Hole” maneuver has very few outliers (0.0-1.2%). For base station distances of up to 96 km, the accuracies for the “Post Hole” maneuver increase to 6-7 cm in both horizontal and vertical components, and increase to 6-13 cm in the horizontal and 29 cm in the vertical for the “Split S-Cuban 8-Split S” maneuver. Due to long periods where less than 5 common satellites were tracked, results for the “9g turns” maneuvers were computed using graceful degradation techniques and are less accurate, emphasizing the need for aided navigation in these extreme cases.

Outliers and decreased accuracy are a result of a combination of effects, including the ability to track sufficient satellites during high dynamic maneuvers, (as seen in the “9g turns” maneuvers), incorrect integer-cycle phase ambiguity resolution due primarily to ionospheric effects, and the strong coupling between multipath, troposphere and vertical parameter estimation [Bock et al., 2000].

Results presented show that with robust data editing, EBE accuracies meet the stringent T&E requirements of 30 cm (RMS) in real-time even at extended ranges from base stations. In these tests, the data were edited manually after the solutions were generated. Robust data editing in real-time is planned for future work. To reduce the number of outliers and improve accuracy we are investigating improved ionospheric modeling and INS/GPS coupling for aided navigation.

These tests provide OSD with sufficient high dynamic performance data to be convinced that the EBE concept deserves serious consideration for deployment in real-time T&E applications.

References